Shortest Path Heuristic

MET CS 526 – Data Structures and Algorithms

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Final Project

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**Introduction**

In many branches of science, including computer science, artificial intelligence, and mathematical optimization, a heuristic is a technique designed for solving a problem more quickly when classic methods are too slow, or for finding an approximate solution when classic methods fail to find any exact solution. This is achieved by trading optimality, completeness, accuracy, and/or precision, for speed. It is basically a shortcut.[[1]](#footnote-1) In this project, we utilize this strategy to implement two shortest path algorithms for traversing a graph from one node to another. A graph is a way of representing relationships that exist between pairs of objects called nodes or vertices, together with a collection of connections between these objects, called edges, which can be weighted or unweighted, and a graph can be overall directed or undirected.[[2]](#footnote-2) In our study, we use an undirected graph. Graphs have many applications, including but not limited to mapping, networking, traffic, circuitry, and the internet. Along with the graph, we also use stacks, heap-based priority queues, hash maps, and hash sets to implement the algorithms. Next, we will take a detailed look at the aforementioned data structures and explain why they were chosen for their specific role in the algorithms.

**Data Structures**

*Graphs:*

A graph is a collection of vertices and edges, so an abstract data type (ADT) to represent a graph would need to provide functions necessary for the creation of, access to, iteration through, removal of, and counting of the vertices and edges in a particular graph. With that in mind, we use the Graph ADT from the text book with the following methods[[3]](#footnote-3):

* **numVertices():** Returns the number of vertices of the graph.
* **vertices():** Returns an iteration of all the vertices of the graph.
* **numEdges():** Returns the number of edges of the graph.
* **edges():** Returns an iteration of all the edges of the graph.
* **getEdge(u, v):** Returns the edge from vertex u to vertex v, in one exists, otherwise return null. For an undirected graph, there is no difference between **getEdge(u, v)** and **getEdge(v, u)**.
* **endVertices(e):** Returns an array containing the two endpoint vertices of the edge e. If the graph is directed, the first vertex is the origin and the second is the destination.
* **opposite(v, e):** For edge e incident to vertex v, returns the other vertex of the edge. An error occurs if e is not incident to v.
* **outDegree(v):** Returns the number of outgoing edges from vertex v.
* **inDegree(v):** Returns the number of incoming edges from vertex v. For an undirected graph, this returns the same value as does out degree.
* **outgoingEdges(v):** Returns an iteration of all outgoing edges from vertex v.
* **incomingEdges(v):** Returns an iteration of all incoming edges from vertex v. For an undirected graph, this returns the same value as does **outgoingEdges(v)**.
* **insertVertex(x):** Creates and returns a new Vertex storing element x.
* **insertEdge(u, v, x):** Creates and returns a new Edge from vertex u to vertex v, storing element x. An error occurs if there already exists an edge from u to v.
* **removeVertex(v):** Removes vertex v and all its incident edges from the graph.
* **removeEdge(e):** Removes edge e from the graph.

There are different ways of implementing the Graph ADT. Including an edge list, an adjacency list, an adjacency map, and an adjacency matrix. The edge list implementation uses an unordered list of all of the edges of the graph. This doesn’t allow for efficient access to a particular edge, or access to all of the edges of a particular vertex. The adjacency list maintains a separate list of each vertex’s edges, allowing for more efficient access to all edges incident a vertex. The adjacency map is similar, but it maintains a map to the edges, with the vertex as the key. This allows access to a specific edge. The adjacency matrix maintains an n x n matrix for a graph with n vertices. The elements of the matrix storing the edges. This allows access to specific edges, but space, while all other implementations use space. The running times of the methods above for each of these implementations are shown in the table below, where n is the number of vertices, m the number of edges, and dv the degree of vertex v.[[4]](#footnote-4)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Method** | **Edge List** | **Adj. List** | **Adj. Map** | **Adj. Matrix** |
| **numVertices()** | O(1) | O(1) | O(1) | O(1) |
| **numEdges()** | O(1) | O(1) | O(1) | O(1) |
| **vertices()** | O(n) | O(n) | O(n) | O(n) |
| **edges()** | O(m) | O(m) | O(m) | O(m) |
| **getEdge(u, v)** | O(m) | O(min(du, dv)) | O(1) exp. | O(1) |
| **outDegree(v)**  **inDegree(v)** | O(m) | O(1) | O(1) | O(n) |
| **outgoingEdges(v)**  **incomingEdges(v)** | O(m) | O(dv) | O(dv) | O(n) |
| **insertVertex(x)** | O(1) | O(1) | O(1) | O(n2) |
| **removeVertex(x)** | O(m) | O(dv) | O(dv) | O(n2) |
| **insertEdge(u, v, x)** | O(1) | O(1) | O(1) exp. | O(1) |
| **removeEdge(e)** | O(1) | O(1) | O(1) exp. | O(1) |

The adjacency map has the best overall running times for these methods, and we will therefore choose this implementation for our graph structure. We also make use of the Vertex and Edge ADT’s which are trivially implemented as interfaces providing only a getElement() method.

*Stacks:*

Stacks are a widely used data structure that adhere to the last-in, first-out principle. This means that objects can be inserted, or pushed, into the stack, and the last element pushed to the stack can be removed, or popped, from the stack. There are numerous applications for these data structures such as the back button on a browser that stores your previously visited sites on a stack, or the undo button in a text editor[[5]](#footnote-5). The Stack ADT used in this study is the Stack provided by Java. This was chosen over the text books implementation because it extends the Vector ADT which has a contains(e) method that executes a linear search of the stack returning true or false depending on if the element was found (to clarify, an linear search isn’t why we chose Java’s stack, the need for a search method on a stack is why we chose it). The contains(e) method runs in worst case time, where n is the number of elements on the stack. All of the other methods, pop(), push(e), peek(), size(), and empty(), run in constant time. This data structure is uniquely suited for backtracking because we have access to the last element on the stack. It is for these push(e) and pop() methods that we chose this data structure. By storing the visited vertices in a stack, we can very efficiently push() a visited vertex to the stack, or check the last vertex visited with the peek() method, and if we need to backtrack, we simply pop() the last visited vertex from the stack. The methods provided by java.util.Stack and the one java.util.Vector used in the project are detailed below.

* **push(e):** Adds element e to the top of the stack.
* **pop():** Removes and returns the top element from the stack (or null if the stack is empty).
* **peek():** Returns but doesn’t remove the top element from the stack (or null if the stack is empty).
* **empty():** Returns a boolean indicating whether the stack is empty.
* **contains():** Returns a boolean indicating whether or not element e is in the stack.

*Heap Priority Queue:*

A Priority Queue is a data structure whose objects are added in any order but are removed based on a given priority. The object whose priority is the lowest will be removed first. The objects are stored in the priority queue as entries, each having a key and a value. There are many applications of priority queues, ‘whales’ at casinos getting the best rooms over lower spending guests, frequent flyers getting access to limited seating, all because they have a better priority. We will need to insert items into this priority queue and then remove the minimum keyed item multiple times in this study, so we will need to implement this with a data structure that allows for us to do this efficiently, enter the heap.

A heap is a binary tree that stores entries at its positions and satisfies the heap-order and complete binary tree property. The heap order property states that in a heap, for every position p other than the root, the key stored at p is greater than or equal to the key stored at p’s parent. And the complete binary tree property states that a heap with height h is complete if and only if the levels 0, 1, …, h – 1 of the tree have the maximum number of nodes possible and the remaining nodes at level h reside in the leftmost possible positions at that level.[[6]](#footnote-6) The result of these properties is that the root will always have the minimum key and the height of the tree will be equal to .

When an entry is added to the heap, it is always added to the left most available position on the bottom level, or the left most position on a new level if bottom level is full. After this, the heap-order property may be violated, so an up-heap bubbling is executed. This is done by comparing the new entry p to its parent q. If the key of p is greater than or equal to the key of q, then nothing must be done. Otherwise, the entries are swapped, and the process is repeated until the heap-order property is satisfied. Because a heap has a max height of , the worst case running time of adding an element to a heap is . We make frequent use of the insert(key, value) method in our study by inserting the adjacent vertices of the current vertex into the priority queue with the direct distance to the destination vertex ( or direct distance + edge weight for algorithm 2) as the key and the vertex as the value.

The other method we make frequent use of is the removeMin() method. Because of the heap-order property, we know that the element with the smallest key value is stored at the root of the heap. We can’t just remove it, or we would have two sub trees. How we implement this method and preserve the heap-order and complete binary tree properties is by replacing the entry at the root with the entry that is at the right most position at the bottom level of the heap and deleting that node. Then perform a down-heap bubbling to preserve the heap-order property. Using the terminology of the last paragraph, there are two cases we must consider, other than the trivial case of the heap only having a root node.[[7]](#footnote-7)

1. If p has no right child, let c be the left child of p.
2. Otherwise (p has both children), let c be a child of p with minimal key.

If the key of p is less than or equal to the key of c, the property is satisfied. Otherwise, if the key of p is greater than the key of c, then we swap the entries stored at p and c and continue with c’s children until the property is restored. Since the height of the tree is , this runs in worst case time. Since we loaded the priority queue with adjacent vertices having the distance to the destination as keys, the vertex we are to travel to next can be accessed in time by calling removeMin(). A description of insert(k, v) and removeMin() follows.[[8]](#footnote-8)

* **insert(k, v):** Creates an entry with key k and value v in the priority queue.
* **removeMin():** Removes and returns an entry (k, v) having minimal key (or null if the priority queue is empty.

*Hash Maps:*

A map is an ADT that allows for the efficient storing and retrieving of values based upon a unique search key. It stores these entries as key value pairs. Since the vertices of our graph are unique, we store the vertex labels as keys and their direct distance to the destination as the values. When implementing a map with a hash table, as we do in this study, we realize an expected get(k) and put(k, v), with worse case being O(n). Other methods that are common in maps are remove(k), size(), isEmpty(), entrySet(), keySet(), and values(). However, we only make use of get(k) and put(k, v), and they are described below. Maps have many applications such as storing student information based on a student ID, domain-name systems map a host name to an IP address, and other use cases that are rather similar.

* **get(k):** Returns the value v associated with key k, if such an entry exists, otherwise returns null.
* **put(k, v):** if the map does not have an entry with key equal to k, then adds entry(k, v) to the map and returns null. Else, replaces the existing value associated with k with v and returns the old value.

*Hash Set:*

The last data structure we will discuss is one we also use quite often but is probably the simplest of all of the data structures we have discussed thus far. A Set is an unordered collection of elements, without duplicates, that should optimally support efficient membership tests. It is similar to a hash map, except that are no associated values. We use this ADT in our study by adding to it vertices we no longer want to visit and checking if a vertex is in the set. The methods provided by Java for this are add(e) and contains(e), both which run in , time. It should be noted that the Hash Set ADT provided by Java, is an unordered set implemented with a hash table. Ordering was not necessary, because we never have to access any of the objects in any order, we just need to add to and check for membership in the set. For these reasons, a Hash Set was chosen to store the vertices we no longer want to visit along our path to the destination vertex. The methods we use are detailed below.[[9]](#footnote-9)

* **add(e):** Adds the element e to the set if it is not already present.
* **contains(e):** Returns a boolean indicating if element e is a member of the set.

Now that we are done discussing the main data structures used in the study, we will turn our attention to the implementation of the heuristic algorithms we employ.

**Algorithm Overview**

The purpose of these heuristic algorithms is to find a shortest path from a source vertex to a destination vertex. And while both of these algorithms accomplish that, it should be noted that the ‘shortest’ path found, may not indeed be ‘the’ shortest path. But, since these are heuristic algorithms, optimality and precision can be shunned for getting any answer, and sometimes even getting the right answer. The first algorithm attempts to find the shortest path to the destination vertex by selecting the next vertex to be the vertex that is adjacent to it and has the shortest, ‘as the bird flies,’ direct distance to the destination vertex. The second algorithm is much like the first, except it chooses the next vertex by choosing the adjacent vertex that has the minimum sum of the edge weight and direct distance to the destination vertex. Both of these values, the direct distance and the direct distance plus the edge distance, are stored as the keys in our priority queue with the vertex and the value, ensuring the proper vertex is selected by calling removeMin(). A high-level overview of each algorithm follows.

**Pseudocode**

**Algorithm:** algorithmOne (g, s, d, m)

**Input:** An undirected AdjacencyMapGraph, g, with weighted edges, a starting point, s, the destination vertex, d, and a map, m, of the vertex labels to their direct distance from the destination vertex.

**Output:** The function returns void but displays to the console the sequence of all vertices along the path taken, the sequence of vertices along the shortest path, and the distance traveled along the edges of the shortest path.

Create Stack ***path*** // to hold shortest path vertices.

Create Stack ***all*** // to hold all vertices visited.

Create HashSet ***dead*** // to hold vertices we no longer want to visit.

Set ***current*** vertex to ***s***.

*Push* ***current*** to ***path*** and ***all***.

*While* ***current*** vertex is not ***d*** do:

Create HeapPriorityQueue ***pq***. // this will contain adjacent vertices as values

Insert vertices adjacent to ***current*** into ***pq***. // and keys are direct distance to ***d***

Set ***next*** vertex to first vertex from ***pq***. // this vertex has shortest direct distance to ***d***

*If* ***next*** vertex is not in ***dead*** and not in ***path,*** then:

Push ***next*** to ***path***.

Push ***next*** to ***all***.

*If* ***next*** has only one adjacent vertex and it is not ***d,*** then:

Add ***next*** to ***dead***. // dead end and we don’t want to go back

*If* ***current*** has only one adjacent vertex then:

Add ***current*** to ***dead***. // dead end and we don’t want to go back

Add ***next*** to ***dead***. // also a dead end

Set ***current*** to ***next***.

*Continue*.

*Else* do:

Remove ***next*** from ***pat***h. // we are back tracking

*If* ***next*** was not the last vertex pushed to ***all*** then:

Push ***next*** to all.

Add ***current*** to ***dead***

Add ***next*** to ***dead***.

Set ***current*** to the last vertex pushed to ***path*** and *continue*;

Calculate the distance traveled along the shortest path.

Display the results.

**Algorithm:** algorithmTwo (g, s, d, m)

**Input:** An undirected AdjacencyMapGraph, g, with weighted edges, a starting point, s, the destination vertex, d, and a map, m, of the vertex labels to their direct distance from the destination vertex.

**Output:** The function returns void but displays to the console the sequence of all vertices along the path taken, the sequence of vertices along the shortest path, and the distance traveled along the edges of the shortest path.

Create Stack ***path*** // to hold shortest path vertices.

Create Stack ***all*** // to hold all vertices visited.

Create HashSet ***dead*** // to hold vertices we no longer want to visit.

Set ***current*** vertex to ***s***.

*Push* ***current*** to ***path*** and ***all***.

*While* ***current*** vertex is not ***d*** do:

Create HeapPriorityQueue ***pq***. // this will contain adjacent vertices as values

Insert vertices adjacent to current into ***pq***. // keys are direct distance to ***d*** + edge weight

Set ***next*** vertex to first vertex from ***pq***. // this vertex has shortest ***d*** + edge weight

*If* ***next*** vertex is not in ***dead*** and not in ***path,*** then:

Push ***next*** to ***path***.

Push ***next*** to ***all***.

*If* ***next*** has only one adjacent vertex and it is not ***d,*** then:

Add ***next*** to ***dead***.

*If* ***current*** has only one adjacent vertex then:

Add ***current*** to ***dead***. // dead end and we don’t want to go back

Add ***next*** to ***dead***. // next is also a dead end

Set ***current*** to ***next***.

*Continue.*

*Else* do:

Remove ***next*** from ***pat***h. // we are backtracking

*If* ***next*** was not the last vertex pushed to ***all*** then:

Push ***next*** to all.

Add ***current*** to ***dead***

Add ***next*** to ***dead***.

Set ***current*** to the last vertex pushed to ***path*** and continue;

Calculate the distance traveled along the shortest path.

Display the results.

**Example Run**

Given the files graph\_input.txt and direct\_distance.txt, **hard coded in class HelperFunctions lines 31 and 89** respectively, construct an AdjacencyMapGraph from graph\_input.txt and a HashMap from direct\_distance.txt. Then the starting vertex is gotten from the user. If the user enters an invalid vertex, the program will loop until a valid vertex is entered. Next, the destination vertex ‘Z’ is set and the first algorithm is executed followed by the second, and the results are displayed to the console. An example output is given below.

*Run One: Run Two:*

Enter a node:

g

Algorithm 1:

Sequence of all nodes: G -> H -> T -> U -> T -> H -> L -> Z

Shortest path: G -> H -> L -> Z

Distance traveled: 359

Algorithm 2:

Sequence of all nodes: G -> H -> T -> U -> T -> H -> L -> Z

Shortest path: G -> H -> L -> Z

Distance traveled: 359

Enter a node:

x

That vertex is not in the graph.

Enter a node:

j

Algorithm 1:

Sequence of all nodes: J -> K -> Z

Shortest path: J -> K -> Z

Distance traveled: 310

Algorithm 2:

Sequence of all nodes: J -> I -> L -> Z

Shortest path: J -> I -> L -> Z

Distance traveled: 278

**Algorithm Analysis**

An overall runtime for these algorithms is out of the scope of this study but we will give it a shot just for fun. We have the main *while* loop, that in the worst case runs through all vertices of the graph except the destination vertex, so worst case is for that loop. Nested in that we have a *for* loop that checks the edges of the current vertex, it runs in time. Inside of that we have the *insert* method of the HeapPriorityQueue that runs in time, and it is executed only when the adjacent vertex is not in the HashSet. Every method after that runs in constant time except the one call to the *contains* method of the shortest path stack, which runs in . We consider the absolute worst-case scenario, where each vertex has an edge to every other vertex, and the previous *next* vertex is further from the destination vertex than the next *next* vertex*.* Also, we will say the stack’s *contains* method is execute n times. In this case, the *while* loop executes times. This also means that the *for* loop executes times, so the *insert* method runs times at , and we add the constant time methods as well, which is somewhere we will give an upper bound of 40, which is fairly generous considering we don’t have to worry about backtracking in this graph. We have:

With this elementary analysis, we can state the algorithm’s worst case as:

**Conclusion**

The two algorithms sometimes take the same path and sometimes take different paths. This is to be expected because they use different selection strategies when choosing the next vertex, and because they are heuristic algorithms, they are not required to always find the correct shortest path. The data structures chosen to solve this problem serve their purpose well. The AdjacencyMapGraph has the all-around best run times of the other graph implementations, and provides numerous functions that allow the creation of, access to, iteration through, removal of, and counting of the vertices and edges in the graph. The HeapPriorityQueue possesses insertion and removal of the minimum element, in our case the closest vertex chosen by either algorithm one or two’s selection strategy. This was better than putting all of the adjacent vertices in a collection and sorting them, because common sorting methods are at best, and then we would still have to retrieve them. The choice of a HeapPriorityQueue for these operations was the best choice. Our choice of a HashMap to store the direct distance to the destination node was also the best choice. It allowed for constant time retrieval of that distance by ‘getting’ the value associated with the vertex label as the key. The HashSet also served us well as we utilized its constant time addition and membership testing. With the amount of times we needed to check for membership in these algorithms, the HashMap played a huge roll in keeping the algorithms efficient. If our worst-case elementary algorithm analysis is correct. For n vertices in a graph, both algorithms will find a ‘shortest’ path in time and utilize space to store the graph.

1. https://en.wikipedia.org/wiki/Heuristic\_(computer\_science) [↑](#footnote-ref-1)
2. Data Structures and Algorithms in Java. 6th edition. Goodrich, Tamassia, Goldwasser. [↑](#footnote-ref-2)
3. Data Structures and Algorithms in Java. 6th edition. Goodrich, Tamassia, Goldwasser. [↑](#footnote-ref-3)
4. Data Structures and Algorithms in Java. 6th edition. Goodrich, Tamassia, Goldwasser.+ [↑](#footnote-ref-4)
5. Data Structures and Algorithms in Java. 6th edition. Goodrich, Tamassia, Goldwasser. [↑](#footnote-ref-5)
6. Data Structures and Algorithms in Java. 6th edition. Goodrich, Tamassia, Goldwasser. [↑](#footnote-ref-6)
7. Data Structures and Algorithms in Java. 6th edition. Goodrich, Tamassia, Goldwasser. [↑](#footnote-ref-7)
8. Data Structures and Algorithms in Java. 6th edition. Goodrich, Tamassia, Goldwasser. [↑](#footnote-ref-8)
9. Data Structures and Algorithms in Java. 6th edition. Goodrich, Tamassia, Goldwasser. [↑](#footnote-ref-9)